Statistical Limits of Fourier Transform Imaging in the γ-ray Energy Range

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Summary

Recent advances in imaging techniques and position-sensitive γ-ray detectors have made feasible hard x-ray and γ-ray telescopes with arc-second resolution [1]. Above an energy of 100 keV, past instrumentation has been limited to a typical angular resolution of a few degrees. A γ-ray imaging device with 1 arc-second resolution would be a dramatic improvement over conventional, non-imaging instrumentation and have substantial new capabilities for observation of astrophysical γ-ray sources. The arc-second γ-ray imager is based on the Fourier transform imaging technique [2]. We briefly describe Fourier transform imaging and its application to hard x-ray and γ-ray imaging. This description is followed by an analysis of Fourier transform imaging in the statistics limited regime. Computer simulations and laboratory demonstrations of practical γ-ray imaging systems are presented.

1. Fourier transform Imaging at Hard X-ray and γ-ray Energies

In recent years, the first imaging γ-ray detectors have been developed for astronomical observations in the energy range 50 keV to 10 MeV. These detectors have used coded-aperture imaging techniques to achieve sub-degree angular resolution [3]. However, in coded aperture-systems the angular resolution is limited by the practically achievable spatial resolution of current photon detectors which is about 1 cm FWHM for 1 MeV γ-rays [4]. For typical coded-aperture systems, the angular resolution is $\sim\tan^{-1}(\Delta x/L)$, where Δx is the spatial resolution of the detectors and L is the spacing between detector and coded aperture mask.

Fourier transform imaging circumvents the limitation imposed by the spatial resolution of the detectors. In this technique [2], a position-sensitive γ-ray detector views a source field through a pair of widely separated collimator grids. The collimator grids consist of parallel slits with a certain orientation ϑ and a certain slit spacing s (see Figure 1). The top and bottom grids differ slightly in either ϑ, or s, or both. The characteristic modulation pattern arises from this small difference in top and bottom grids. Each pair of collimator grids allows the measurement of the phase and amplitude of a particular Fourier component of the source angular distribution. In analogy to radio astronomy, each pair of collimators provides one "baseline" for measurement of the angular source distribution. The phase and amplitude information from the individual γ-ray measurements is then combined to form images using algorithms identical to those of radio astronomy.

Practical implementation of the Fourier transform technique at γ-ray energies requires large-area position sensitive γ-ray detectors. We have been active in the development of such detectors [4]. A FWHM position resolution of 0.7 cm over 600 cm^2 was obtained, making it suitable for modulation detection techniques. We are currently collecting data with a position-sensitive NaI scintillation camera and a set of 7 pairs of Fourier transform grids. Modulation patterns and images obtained from this laboratory demonstration will be discussed.

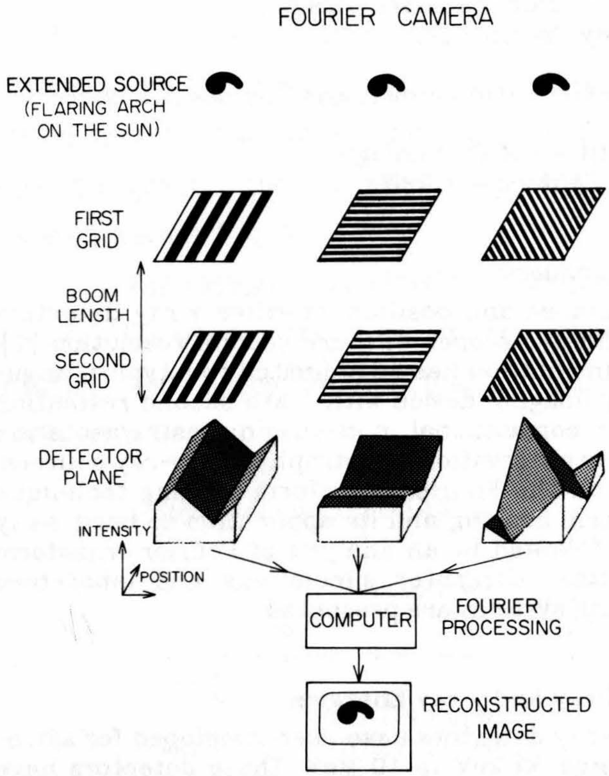

Figure 1.

2. Statistical Limits in Fourier Transform Imaging

We are in the process of a detailed analysis of the statistical limits of Fourier transform imaging. Whereas in multiple baseline radio astronomy the quantum limit is never approached, in γ-ray astronomy the low photon flux and high background counting rates require a careful statistical analysis of imaging performance.

To analyze these problems, consider a single point source. The distribution of counts in the detector can be approximated by:

$$N(\vartheta)=\frac{N}{2\pi P}\left[1+A\cos(\vartheta-\varphi)\right]$$

where ϑ is the position on the detector plane, A is the signal to noise ratio, N is the total number of counts, $N(\vartheta)$ is the number of counts per unit length on the detector, P is length of one radian of modulation pattern, and φ is the angular position of the source multiplied by a scaling constant. Now let us consider the Fourier integral of image intensity with respect to position:

$$V=P\int_{-\pi}^{\pi}N(\vartheta)e^{i\vartheta}d\vartheta$$

If we graph this on the complex plane, we find the complex exponential argument of V is the phase position of the peak and its magnitude is one-half the total number of counts times the signal-to-noise ratio. If we plot the whole chain of events (i.e. plot the contribution to V on an event-by-event basis), the plot gives a random walk (Figure 2).

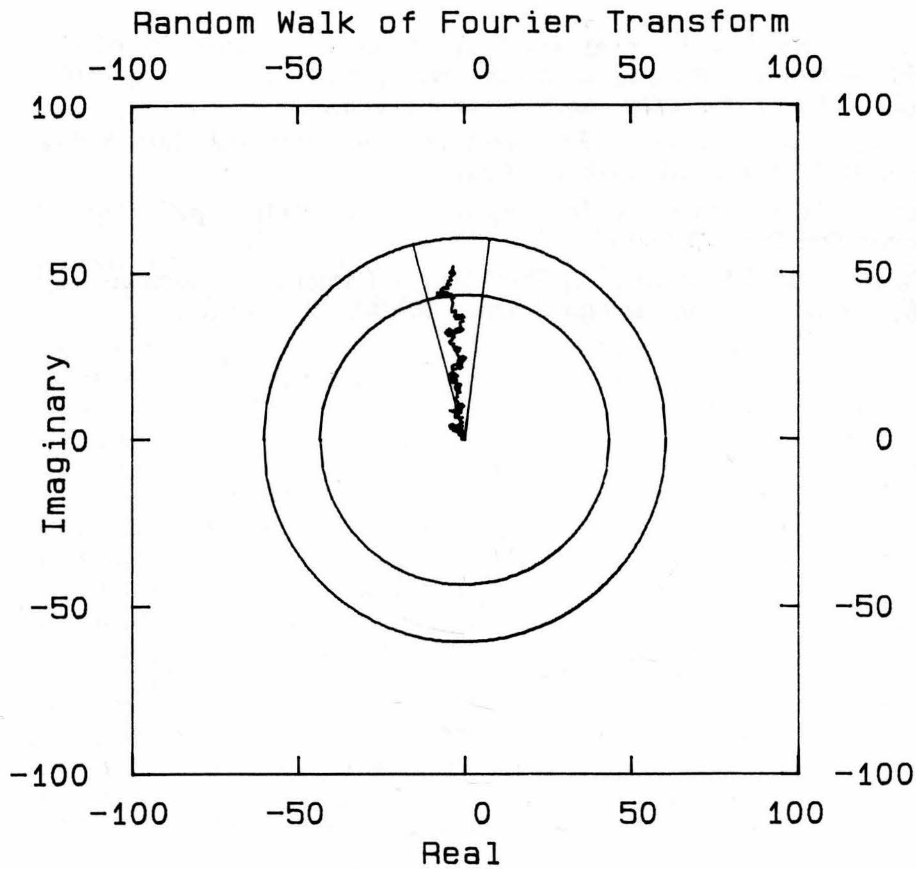

Figure 2.

From the distribution of N(ϑ) and the random walk of V, the following equations for the standard deviation of φ and A can be derived for large N.

$$\sigma(\varphi)=\frac{1}{A}\sqrt{(\frac{2}{N})}, \quad \sigma(A)=\sqrt{(\frac{2}{N}(1-\frac{A^2}{2}))}$$

The wedge shaped region in Figure 2 gives $\sigma(\varphi)$ and the concentric circles form the region for σ(A). Each of these regions has a 68% confidence of the actual value being located in the region.

The analysis summarized above is applicable to a single collimator grid pair and assumes N large. We are currently analyzing the statistical errors in combining the measurements of multiple collimator grid pairs to determine the angular position of a point source in one and two dimensions. We will also treat the problem of errors in resolving two point sources.

Because basic information from a Fourier transform γ-ray telescope (i.e. phase and amplitude) is the same as that collected from multiple baseline radio interferometers, we have applied software algorithms developed for radio astronomy to simulated γ-ray measurements. We will present the results of these simulations with emphasis on the case of low statistics. These simulations will address the question of the number of collimator grid pairs needed to achieve a given image accuracy.

References

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